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An empirical study of the predictive utility of an exponential progress function

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OF AN EXPONENTIAL PROGRESS FUNCTION

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ABSTRACT

Manufacturing progress functions of the basic form not unlike that as originally conceived three decades ago have been used as cost forecasting models for manufacturing processes with wide acceptance within the airframe industry over the intervening years and, more recently, in industry in general. A new exponential progress function, established through a theoretical development, is considered as a means to improve the forecasting utility of the progress function approach to manufacturing process cost estimation.

Empirical cases involving a total of eighteen operators at two separate manufacturing facilities are presented to show that the conventional form of the progress function provides a significantly better portrayal of operator progress than does the exponential form, especially during the initial phase of the learning process. Extensions of the theory of the exponential function are shown to be useful in evaluating the factors which influence the rate of progress. It is concluded that the exponential progress function is not a general expression portraying operator progress. However, no evidence is presented to disclaim the validity of the function in applications not meeting the conditions of the present study.

INTRODUCTION

The concept of the manufacturing progress function has commanded varying degrees of acceptance by the various manufacturing industries during the function's thirty years of existence. The airframe industry, within which the manufacturing progress function was conceived and nurtured to its "Saturnis regne", has been the prime proponent and user of this cost-quantity relationship as a forecasting device. Introduced by Wright[1] in the mid 1930's to the airframe industry, the manufacturing progress function found few advocates outside that industry until the early 1950's when empirical evidence of the generality of cost-quantity relationships to manufacturing processes was advanced.

A study of manufacturing progress functions by Hirsch[2] was one of the first articles of major significance to the manufacturing industries in general. Andress[3], in a management-oriented article two years later, covered briefly the theory of the manufacturing progress function and urged consideration of the function for possible applications in more diverse industries. He suggested electronics, home appliances, residential home construction, shipbuilding, and machine shops as representative areas of industry which should be able to apply the function profitably. Andress, however, concluded that some industries would

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[1]Wright, T. P., "Factors Affecting the Cost of Airplanes," Journal of the Aeronautical Sciences, Vol. 3, No. 4, Feb., 1936, pp. 122-128.

[2]Hirsch, W. Z., "Manufacturing Progress Functions," The Review of Economics and Statistics, Vol. 34, May, 1952, pp. 143-155.

[3]Andress, F. J., "The Learning Curve as a Production Tool," Harvard Business Review, Vol. 32, No. 1, Jan.-Feb., 1954, pp. 87-97.

find the function of little value, citing basic chemicals, plastics and petroleum refining as specific cases.

Searle[4], possible unknown to Andress, had provided evidence of the applicability of the function in shipbuilding several years earlier using data derived from World War II activity; however, Searle did not recognize the manufacturing progress function concept per se and even concluded his work with the thought that post-war shipbuilding would not show the same cost-quantity relationships because of the cutbacks in the activity of the ship yards. Andress's insight into cost-quantity relationships proved to be slightly more narrow than reality when Hirschmann[5], writing in the same journal twelve years later, showed the applicability of the function in petroleum processing. He further advanced evidence of its usefulness in plant maintenance activity, heavy equipment construction, the electric power industry and the basic steel industry.

In addition to the works by Andress and Hirschmann, studies conducted and reported by Conway and Schultz[6], Cochran[7], and Williams[8]

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[4]Searle, A. D., "Productivity Changes in Selected Wartime Shipguilding Programs," Monthly Labor Review, Dec. 1945, pp. 1132-1147.

[5]Hirschmann, W. B., "Profit From the Learning Curve," Harvard Business Review, Vol. 42, No. 1, Jan. - Feb., 1964, pp. 125-139.

[6]Conway, R. W., and Schultz, A., Jr., "The Manufacturing Progress Function," The Journal of Industrial Engineering, Vol. X, No. 1, Jan. - Feb., 1959, pp. 39-54.

[7]Cochran, E. B., "New Concepts of the Learning Curve," The Journal of Industrial Engineering, Vol. XI, No. 4, July-Aug., 1960, pp. 317-327.

[8]Williams, P. F., "The Application of Manufacturing Improvement Curves in Multi-Product Industries," The Journal of Industrial Engineering, Vol. XII, No. 2, Mar. - Apr., 1961, pp. 108-112.

have done much to establish the fact that manufacturing progress functions are not the undivided domain of the airframe industry.

As introduced by Wright, the manufacturing progress function was given by the relationship of:

$$\overline{y} = ax^b \tag{1}$$

where

x = cumulative number of product units produced,

$\overline{y}$ = cumulative average labor hours per product unit required to produce the first x units,

and a and b are parameters[9].

A more common form of Wright's innovation (the cumulative average curve) is that of Crawford's unit curve modification[10]. This function is given by the relationship of:

$$y = a'x^{b'} \tag{2}$$

where

x = cumulative number of product units produced,

y = labor hours required to produce the xth cumulative product unit,

and a' and b' are parameters.

As to the choice of one of these models for usage, Conway and Schultz contend that:

> Since proponents of neither model are able to establish their position by logic, and empirical evidence is far from sufficient to establish the superiority of one alternative, the choice in usage has been largely a matter of computational convenience[11].

[9]Wright, T. P., op. cit., pp. 124-125.

[10]Crawford, J. R., Learning Curve, Ship Curve, Ratios, Related Data, Lockheed Aircraft Corporation, Burbank, California, (n.d.).

[11]Conway, R. W., and Schultz, A., Jr., op. cit., p. 41.

Because the manufacturing industries, with the exception of the airframe industry, have found the unit curve of more interest for their forecasting requirements, the latter model has the more universal audience among users of manufacturing progress functions.

Since, in both models, the b parameters are always taken as negative quantities, the manufacturing progress functions are basically negative power functions. In order to facilitate the use of these models computationally, a conversion to logarithmic coordinates is usually made, yielding functions which have the appearance of straight lines. In these transformed functions, the b parameters are interpreted as the slopes of the straight lines and the logarithms of the a parameters are associated with the intercepts on the ordinate. This characteristic of the transformed functions, i.e., their linearity, has given rise to "the linear hypothesis" among users of the manufacturing progress functions[12]. Almost universally, this basic assumption of a linear[13] relationship of unit man-hour cost and cumulative output is made when applying the cost-quantity relationship.

The manufacturing progress function[14], with its related linear hypothesis, predicts that the rate of output will increase indefinitely as the production process continues. Figure 1 illustrates a hypo-

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[12]Asher, H., Cost-Quantity Relationships in the Airframe Industry, Report R-291, The RAND Corporation, Santa Monica, California, July 1, 1956, p. 67.

[13]Although implied in the previous sentence, linearity as used in this context should be defined as meaning linear on logarithmic coordinates.

[14]Since the unit curve appears to have a wider range of adherents within the manufacturing industries, this model will be referenced throughout the following discussion.

thetical case for which the manufacturing progress function is plotted on logarithmic coordinates. As may be seen from the illustrated hypothetical situation, the trend of lower man-hour cost per unit continues as long as production continues. Mathematically, this characteristic may be shown by finding the equation for cumulative man-hours $\underline{t}$ required for the production of the first $\underline{i}$ units and differentiating this equation for the rate of output $\frac{di}{dt}$; e.g.:

$$y = ax^{-b}, \quad 0 < b < 1, \tag{3}$$

$$t = \int_0^i ax^{-b}dx = \frac{ai^{1-b}}{1-b}, \tag{4}$$

$$\frac{dt}{di} = ai^{-b}, \tag{5}$$

$$\frac{di}{dt} = \frac{i^b}{a}, \quad 0 < b < 1. \tag{6}$$

From (6), it is evident that manufacturing progress function predicts that the rate of output $\frac{di}{dt}$ continues indefinitely at a diminishing rate as production cumulates.

One writer, discussing his analysis of manufacturing progress curves derived from information of World War II airframe production, states:

> It is indisputable that lower direct labor costs occur as the number of items produced increases; the evidence on this point is overwhelming. Questions can be raised, however: (1) How long does this reduction continue? (2) Can it be represented by a linear function on double log scale?---[15]

In answering the first question, he concludes that:

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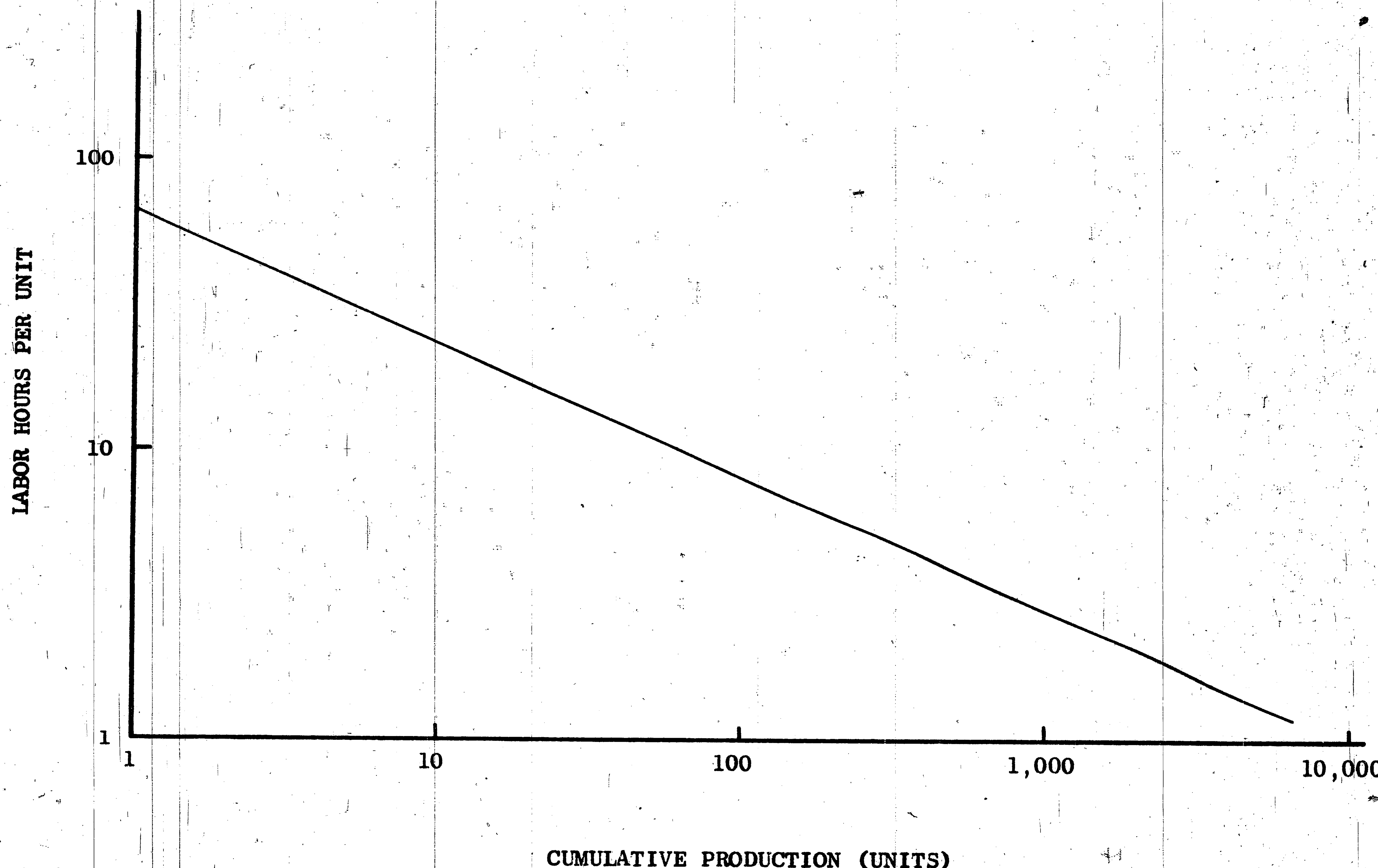
[15] Alchian, A., "Reliability of Progress Curves in Airframe Production," Econometrica, Vol. 31, No. 4, October, 1963, p. 680.

FIGURE 1. HYPOTHETICAL MANUFACTURING PROGRESS FUNCTION OF THE FORM $y = 71x^{-0.474}$



> In every case there was no evidence of any cessation of a decline. This conclusion is based on visual examination of the graphs presented in the Source Book[16]. No elaborate statistical analysis appears to be needed to answer this question, given the available data. Whether or not the decline would cease for substantially larger N[17] could not, of course, be determined[18].

And as to the second question, he observes that:

> Since it appeared that the observations would not be sufficient to give a very powerful test of the linear hypothesis with respect to some acceptable alternative, it was believed best to postpone such possible tests until more adequate observations were available. For the rest of this study linearity is simply postulated[19].

A second writer, drawing data from the Aeronautical Manufacturers Planning Reports (AMPR's)[20] and supplementing this information with data directly from the airframe manufacturers, states that:

> It is safe to conclude, on the basis of the admittedly limited sample examined in this study, that the conventional linear progress curve is not an accurate description of the relationship between unit cost and cumulative output. Beyond certain values of cumulative output, both the labor - and the production-cost curves develop convexities[21].

The writer identified the "certain values of cumulative output" as 300 units for the data at his disposal.

Several other writers have reported varied cases in which the

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[16]This is a reference to the Source Book of World War II Basic Data; Airframe Industry, Vol. 1, prepared by the AAF Material Command, Wright Field (undated).

[17]N refers to cumulative production quantity.

[18]Alchian, A., op. cit., p. 683.

[19]Ibid.

[20]AMPR's were the documents from which the Source Book was derived. Alchian, above, notes this fact in his work and states: "The reliability of the AMPR's has been subject to a good deal of speculation and remains a moot point."

[21]Asher, H., op. cit., p. 129.

progress curves develop convexities, or "leveling-offs".[22,23] Usually, these writers have asserted that there is an assignable cause for the "leveling-off" of the progress curve; however, no one has been able to relate this phenomenon in terms of quantitative cause and effect.

Asher[24] has reported a study made of the results of production of twelve types of post-World War II fighter aircraft in which the relationship between man-hour cost of unit number one and progress curve slope was investigated. As a result of this study, which included six models that were new to the producing facilities and six were, in many respects, similar to aircraft previously produced by the same facilities, he found that:

> ---in the case of new models, relatively high unit number one costs were experienced, resulting initially in rather steep progress curves; whereas in the case of old models, relatively low unit number one costs were experienced, resulting initially in flat progress curves[25].

The most interesting implication of this study comes from the result of the correlation analysis of the empirical observations relating to the above relationship. Using the best-fit equation for slope in terms of unit number one cost from this study, Asher obtained a family of hypothetical progress curves to unit 300 (no empirical observations beyond this point were considered in the analysis of the twelve original fighter progress curves). The plot of this family of curves is illus-

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[22]Conway, R. W., and Schultz, A., Jr., op. cit.

[23]Hirschmann, W. B., op. cit., pp. 126-127.

[24]Asher, H., op. cit., pp. 76-82.

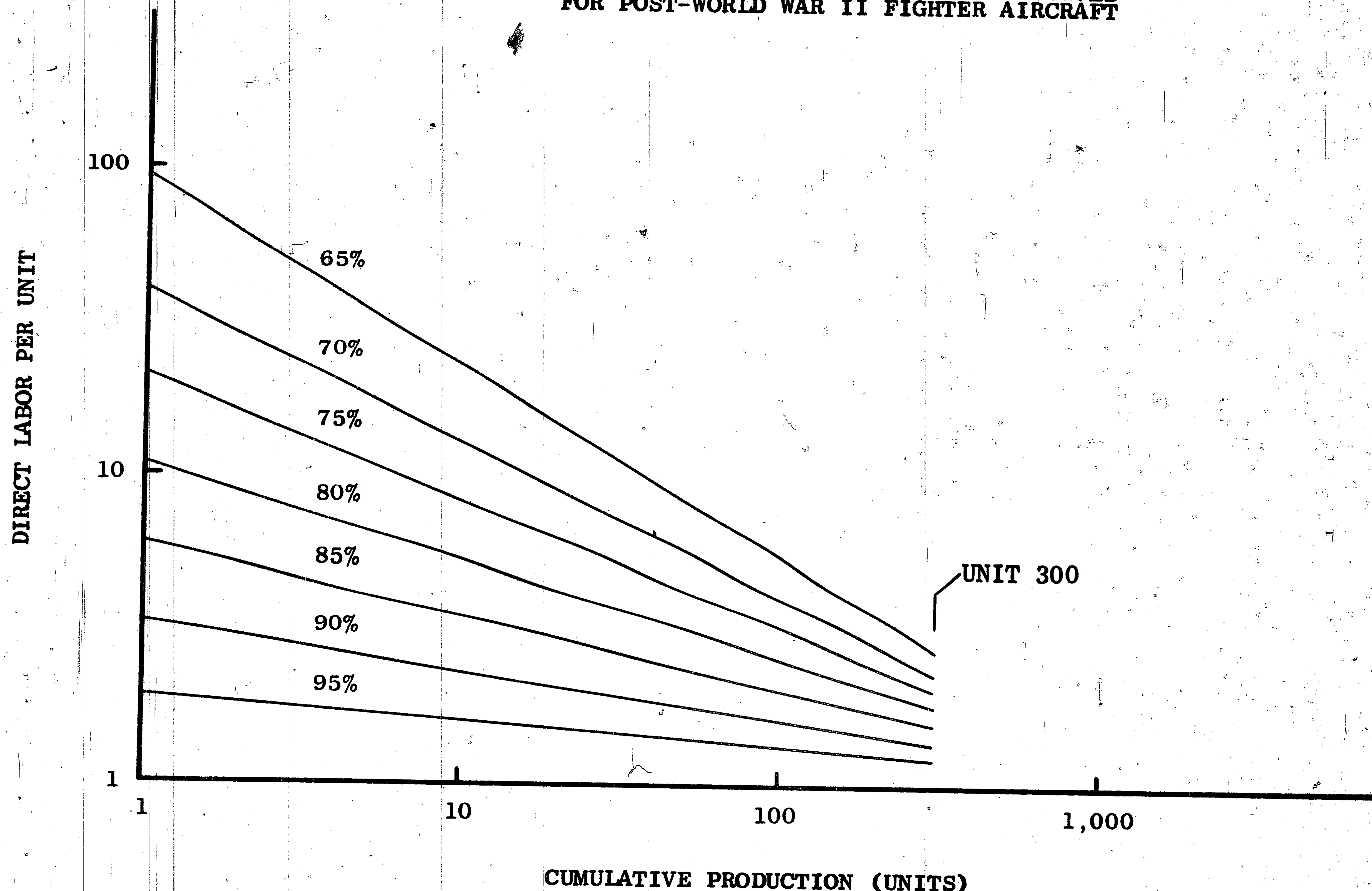
[25]Asher, H., op. cit., p. 80.

trated in Figure 2. If the progress curves in this figure were all linear beyond unit 300, they would eventually intersect. This would imply that after the point of intersection was passed, an airplane produced along a steep curve would eventually be produced for fewer man hours than an airplane that was initially familiar to the producing facility. The probability of this occurrence is rather remote, although not entirely impossible. Thus, if the curves are to avoid a point of intersection, they must become either convex or concave. All empirical evidence tends to discredit the development of concavities in these curves. This reasoning suggests that there is some minimum man hours that is the asymptote of the family of curves or at least several asymptotes for certain groupings within the family of curves.

There are two possible explanations for the discrepancy between the findings of the several writers who have reported the convex tendencies of progress curves in general and the apparent linearity of the progress curves for many models of product in the airframe industry, particularly those models produced during World War II:

(1) Imposed Control Objectives: Once a control or quantitative objective has been imposed on an organization, it seems almost inherent that strong forces are created within that organization to make its performance fit the objective previously set down. Thus, the use of the manufacturing progress function as a control device, which has been the case for the vast majority of the programs within the airframe industry, will inevitably influence the production data from these programs. Hence, the argument from the

FIGURE 2. FAMILY OF DIRECT LABOR PROGRESS CURVES FOR POST-WORLD WAR II FIGHTER AIRCRAFT



airframe industry that the linear function has "worked" in practice is at best a weak defense of the linear hypothesis.

(2) Accelerated Technological Progress: With the advent of World War II, the airframe industry was called upon to increase its annual production from approximately 3,100 aircraft in 1940 to a level that had reached approximately 81,000 units in 1944.[26] To accomplish this unprecedented increase in annual output, the airframe industry adopted many new and efficient methods of production. This dramatic increase in production with its related necessity for changes to more efficient production techniques could have important implications for manufacturing progress function concepts.

If it is assumed that a minimum man-hour cost exists for a particular model with a given method of production, each successive improvement in production technique would have the effect of reducing the minimum cost for that model. Conceivably, after each lowering of the minimum labor cost, the progress curve for that model might be expected to change direction slightly and move toward this new minimum value.

A hypothetical case is illustrated in Figure 3. A change in the methods of production at unit 15 has the effect of lowering the minimum cost from 10 man hours, which was established by the production methods adopted to build the first several units, to 6 man hours. Thus, rather than proceeding along the solid curve (curve 1), the progress curve moves to the dashed curve (curve 2).

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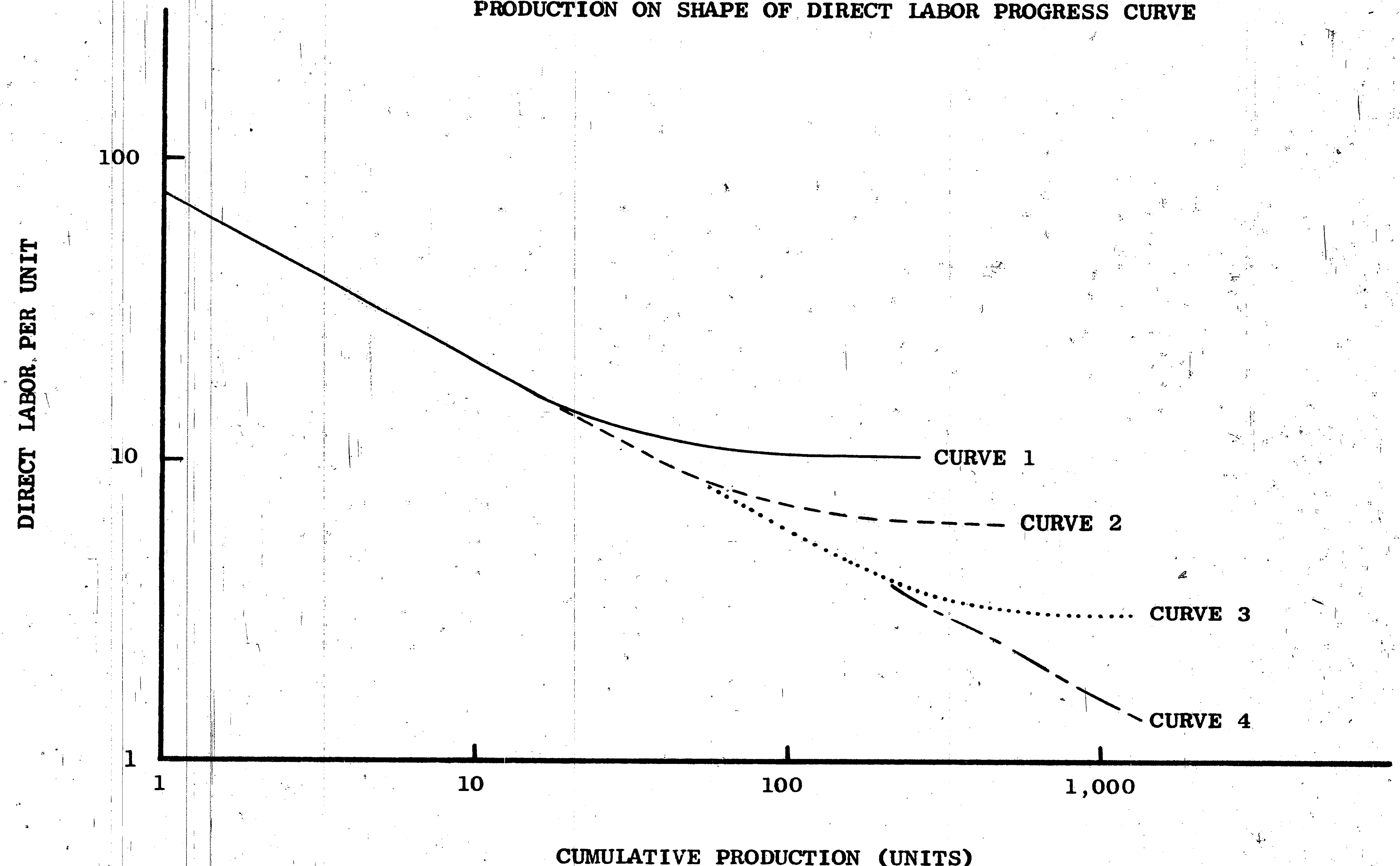
[26]Asher, H., op. cit., p. 130.

FIGURE 3. HYPOTHETICAL EXAMPLE OF THE EFFECTS OF CHANGES IN METHODS OF PRODUCTION ON SHAPE OF DIRECT LABOR PROGRESS CURVE

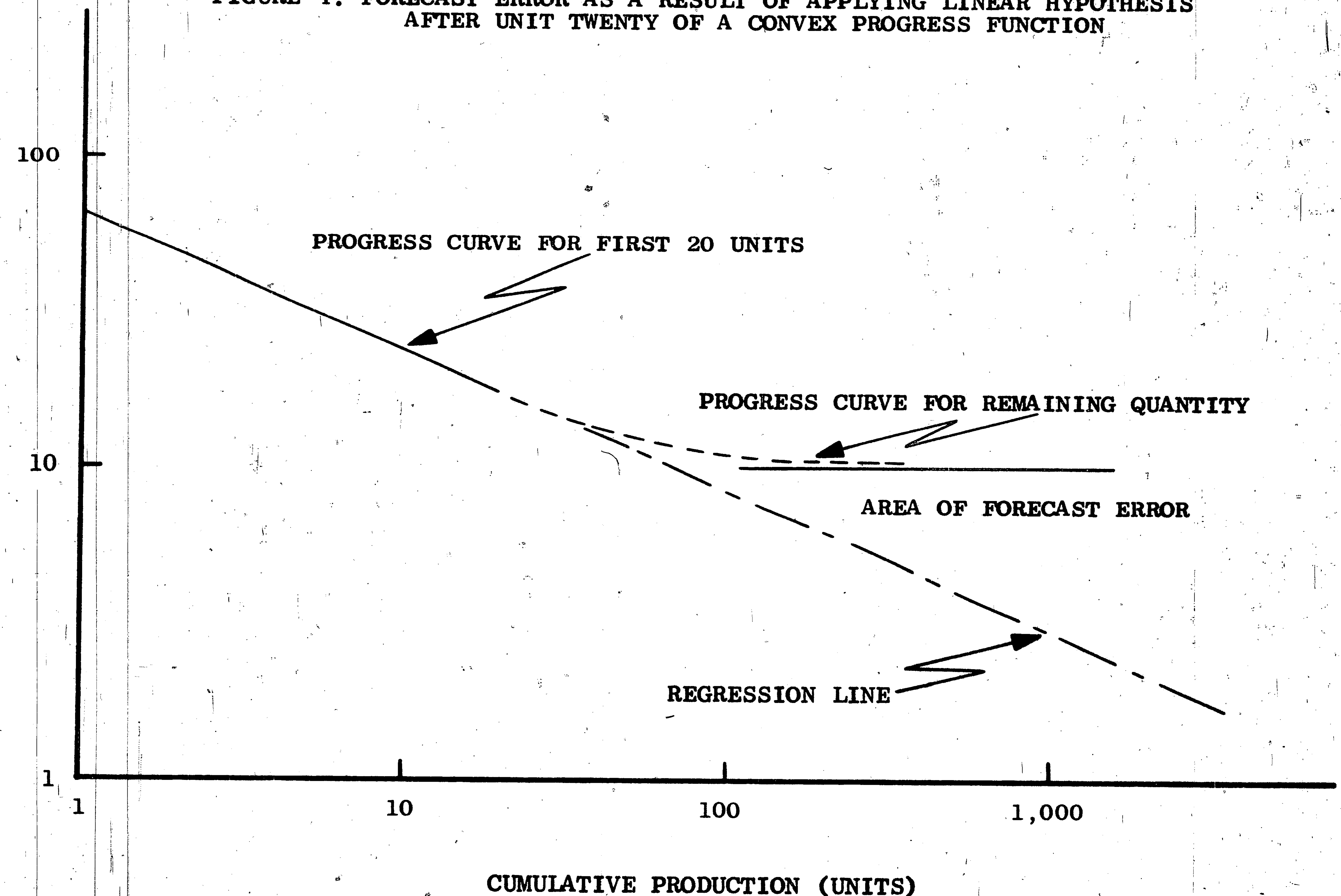


If another change in the methods of production is introduced at unit 45, the progress curve may now have a minimum value of 3 man hours and thus move to the dotted curve (curve 3). In this hypothetical case, the progress curve actually recorded would be the one obtained by eliminating that part of each curve that follows a change in the methods of production. It is apparent that the resulting curve is closely approximated by a linear function. For example, if the data obtained from the production of the first 150 units were used for fitting a regression curve, the conventional manufacturing progress function would provide an excellent fit to the data. The broken curve (curve 4) following unit 150 is the resulting regression curve.

As an illustration of the type and amount of error that could be generated by the acceptance of the linear hypothesis when in reality the progress curve develops a convexity, Figure 4 shows an exaggerated hypothetical case which represents the situation. If, after only twenty units of the product have been manufactured, the preliminary results are used as empirical data for a fit of the conventional manufacturing progress function for forecasting purposes, the broken regression line shown in Figure 4 would result. Then, assuming the dashed convex curve as being the progress curve for the remaining quantity of production, the difference between the dashed and the broken curves represents the amount by which the true progress curve has been under-estimated.

From the discussion of the discrepancy between the convexities which develop in progress curves for varied cases studied by several

FIGURE 4. FORECAST ERROR AS A RESULT OF APPLYING LINEAR HYPOTHESIS AFTER UNIT TWENTY OF A CONVEX PROGRESS FUNCTION



writers and the linear hypothesis of the conventional manufacturing progress function, another problem area emerges which is probably more basic than the fact that the conventional form of the function does not appear to be an accurate description of the cost-quantity relationship. This problem is that the conventional manufacturing progress function is strictly an empirical phenomenon with no underlying theory. Thus, in its present form, the function tells nothing about the factors that may influence the progress rate associated with the function.

Two sentences from the conclusions of one of the most referenced articles on the conventional manufacturing progress function seem to epitomize the problem areas very effectively. First:

> ---the problem of convexity and its resultant errors is still unsolved,---[27]

And, secondly:

> ---caution must be exerted in any application of the function to base it firmly upon empirical studies within the firm or plant where it is to be applied---[28]

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[27] Conway, R. W., and Schultz, A., Jr., op. cit., p. 53.

[28] Ibid.

STATEMENT OF THE PROBLEM

The members of the general class of functions introduced as manufacturing progress functions have been denoted variously as manufacturing improvement curves, aircraft progress functions, aircraft learning curves and industrial learning curves among several other designations. Despite the somewhat common usage of the term, some writers are hesitant to apply the designation "learning" to the phenomenon expressed by the relationship of labor hours and cumulative production[1]. However, if learning is given the expanded meaning of acquisition and improvement of skills as measured by increased productivity (whether the effort for the accomplishment be made by an individual, a group of individuals engaged in common production activities or in different activities involved with a common product, and/or an entire industry), the concept of the manufacturing progress function may be visualized then as one of learning.

The applicability of the concept of learning to the area of manufacturing progress functions seems even more appropriate when considering the common form of the expressions for conventional industrial learning curves in the limited sense for both individual and group learning over a range[2] and for conventional manufacturing progress functions[3]; i.e.:

---

[1]c. f., Conway, R. W., and Schultz, A., Jr., op. cit., p. 42.

[2]Kilbridge, M. D., "A Model for Industrial Learning Costs," Management Science, Vol. 8, No. 4, July, 1962, p. 521.

[3]Hirschmann, W. B., op. cit., p. 128.

$$y = ax^b. \tag{7}$$

One writer states that:

> The industrial learning curve thus embraces more than the increasing skill of an individual by repetition of a simple operation. Instead, it describes a more complex organism--the collective efforts of many people, some in line and others in staff positions, but all aiming to accomplish a common task progressively more efficiently[4].

This statement provides an excellent example of the expanded definition of learning which is used in the remainder of this thesis.

With this introduction of an expanded definition of learning, the usage of the term "manufacturing progress function" will be minimized and the designation "industrial learning curve" will be employed instead to describe the same phenomenon. To eliminate any misunderstanding which might arise, the former nomenclature was used in the introductory material only because of its prevalence in some of the more current literature[5].

It seems appropriate at this time to introduce another concept used later in this thesis which departs from the uniformity already established in the field. Usually the cardinal measure of "progress" in terms of manufacturing improvement is decreasing man hours per unit of product. An alternative measure would be the inverse of this, i.e., increasing units per man hour, or rate of output. The latter measure is used in this thesis for reasons which become obvious later in this section. This alternative measure in no way detracts from the validity of the empirical study presented in the following sections.

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[4]Hirschmann, W. B., op. cit., p. 128.

[5]The introduction of an expanded definition of learning was deferred for the same reason.

Thus, in the empirical study the conventional industrial learning curve will have the form of (7), where the parameter b will have the same range as given in (6)[6].

Levy[7] has shown the theoretical development of an exponential learning[8] function, or as designated by the originator, "an adaptive function," which is the result of an attempt to produce solutions to certain problem areas involved with the concepts of the conventional manufacturing progress function. These two problem areas are identical to those discussed in an earlier section of this thesis. The following paragraphs will show the essence of the development procedure for deriving the exponential learning function.

A firm undertaking the operation of a new process has a maximum rate of output Y which it desires to achieve to effect the most economical operation of this process. The firm may or may not be consciously aware of the quantitative value of this limiting parameter and, if so, the knowledge of this value is usually at best only a rough estimate of some true value; however, these shortcomings do not eliminate the fact that this maximum rate of output does exist and that the estimate can prove useful. This maximum rate of output depends upon the rates of inputs to the process, machine capacities, state of the art, monetary

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[6]The concept of rate of output as a measure was given in the Introduction. Equation (6) gives the relationship which will be used in the empirical study; however, this relationship will be presented in the form illustrated by (7).

[7]Levy, F. K., Adaptation in the Production Process, Graduate School of Business, Stanford University, Stanford, California, February, 1964, (unpublished paper), 32 p.

[8]At the risk of being repetitious, it is again stressed that the work "learning" is defined here by its expanded meaning.

considerations, and other limitations imposed by factors beyond the immediate and direct control of the firm. Due to the learning process involved in the performance of any activity, the firm begins the operation at some lesser rate of output $\underline{y}$ and approaches $\underline{Y}$ through this learning process.

It is assumed that the firm's "stock of knowledge" on a particular process at any specified time can be summarized in the stock of the product it has produced by that process up to that time. Thus, as the firm produces more of that given product, it increases its "stock of knowledge" on that product and increases its rate of output. Then $\underline{y(x)}$ represents the rate of output $\underline{y}$ after $\underline{x}$ units have been produced.

Because the firm begins the operation of the process at some rate of output $\underline{y}$ and approaches $\underline{Y}$, at any point of production

$$y(x) < Y \; ; \tag{8}$$

also, the difference in the rate of production between the $\underline{x}$th and the ($\underline{x}$ + 1)st unit of product is given by

$$y(x + 1) - y(x) \geq 0 \; . \tag{9}$$

The maximum improvement possible after $\underline{x}$ units have been produced is

$$Y - y(x) \; . \tag{10}$$

The rate of "adaptation" by the firm to the maximum rate of output after $\underline{x}$ units have been produced is defined by

$$b(x) = \frac{y(x + 1) - y(x)}{Y - y(x)} \; . \tag{11}$$

This may be stated as:

$$y(x + 1) - y(x) = b(x) \left[Y - y(x) \right] \; . \tag{12}$$

Replacing the finite difference to the left of the equality in (12) by its continuous approximation,

$$\frac{dy(x)}{dx} = b(x)\Big[Y - y(x)\Big] \tag{13}$$

results and solving (13) for y(x) yields

$$y(x) = Y\Big\{1 - e^{-[c + b(x)x]}\Big\}, \tag{14}$$

where $\underline{c}$ is a constant of integration.

The parameter $\underline{b(x)}$ will depend upon those variables which influence the firm's rate of "adaptation" to the maximum rate of output. These variables are assumed to be exogenous of $\underline{x}$ and are denoted by $\underline{z}_1$, $\underline{z}_2$,--- $\underline{z}_n$, so that

$$b(x,z) = f(z_1, z_2, \text{-----}, z_n). \tag{15}$$

Assuming that $\underline{b(x, z)}$ is at least twice differentiable, (15) may be expanded by a Taylor Series and, by dropping all terms higher than first order,

$$b(x,z) = \alpha_0 + \sum_{i=1}^{n} \alpha_i z_i \tag{16}$$

results. This expression may be restated as:

$$b(x,z) = \mu, \tag{17}$$

where

$$\mu = \alpha_0 + \sum_{i=1}^{n} \alpha_i z_i. \tag{18}$$

Substituting the value of $\underline{b(x)}$ given by (17) in (14),

$$y(x) = Y\Big[1 - e^{-(c + \mu x)}\Big] \tag{19}$$

results. From (19), it may be seen that as the firm accumulates experience (i.e., $\underline{x}$ increases) its rate of output approaches the desired rate $\underline{Y}$. Further, by the relationship given in (18), the rate of "adaptation" may be related to the variables that affect it, i.e., the $\underline{z_i}$'s.

If certain types of techniques deliberately applied by the firm reduce its rate of "adaptation" to the desired rate of output, they must be of such a nature that they enable it to begin at a higher level of efficiency than possible without them and to maintain this higher efficiency as production continues. This implies that $\underline{y(0)}$ would be greater with the application of these techniques than without it. From (19),

$$y(0) = Y(1 - e^{-c}) \; ; \tag{20}$$

therefore, for $\underline{y(0)}$ to be greater with the application of these techniques, the term $\underline{c}$ must be assumed as an increasing function of those factors which influence the firm's initial efficiency on a process. Assuming that increases in any of these factors change $\underline{c}$ at a decreasing rate,

$$c = \beta_0 \prod_{i=1}^{n} z_i^{\beta_i} \, , \tag{21}$$

where

$$0 < \left|\beta_i\right| < 1 \, , \tag{22}$$

may be postulated as a functional form for purposes of estimation.

Those variables $\underline{z_i}$ which influence the rate of progress $\underline{\mu}$[9] and which determine the initial efficiency $\underline{c}$ have been postulated as being related to $\underline{\mu}$ by (18) and to $\underline{c}$ by (21). To aid in the identification of

[9]Previously, $\underline{\mu}$ was defined as being the rate of "adaptation" to the maximum rate of output $\underline{Y}$. Rate of progress will be used synonymously with the rate of "adaptation".

these variables, learning, as defined by the expanded meaning, may be divided into two main categories: (1) that which results from techniques applied to the process, or induced learning[10]; and (2) that which results from the activities of the employee involved directly in the process, or operator learning. Each of these categories may be further subdivided according to various characteristics.

(1) Induced Learning: Induced learning is that improvement which results from techniques applied to the process both prior to and during the operational activities of the process. Thus, induced learning may be broken down into pre-production planning activities and production improvement activities. Pre-production activities include such factors as equipment and tool selection, magnitude of product design effort for manufacturability, jig and fixture design, work methods analysis, and shop organization. After the operation of the process begins, changes in tooling, methods, jigs and fixtures, and product design; improved management techniques; and incentive pay plans are common efforts to improve production rates.

Induced learning may be brought to bear through a process which is not necessarily a matter of unadulterated planning. The knowledge of the technique which is applied for improvement may have been gained through chance from the process's environment. One writer has identified the unexpected acquisition of knowledge resulting in action for improvement as "random learning"[11], while another

[10]Induced learning is a term employed by F. K. Levy (op. cit., p. 7); however, it is used here to include more than that which is delineated by Levy in his work

[11]Levy, F. K., op. cit., p. 8

terms it "progress by serendipity"[12].

Most of these factors have been elaborated by several writers in dealing with industrial learning curves[13]; therefore, it is superfluous to the purpose of this thesis to pursue these matters beyond the recognition of their existence and their relationship to the rate of progress $\underline{\mu}$ and the initial efficiency $\underline{c}$. It is generally acknowledged that the amount of pre-production planning activity is related inversely to the rate of progress $\underline{\mu}$ but directly to the firm's initial efficiency $\underline{c}$ and that the amount of production improvement activity is related directly to the rate of progress $\underline{\mu}$.

(2) Operator Learning: Operator learning is that improvement which results from the activities of the employee involved directly in the operation of the process. Operator learning may be subdivided conceptually into two classes: (1) the learning process in the limited sense of learning; i.e., the process of becoming more adept at performing an assigned task through repetitive performance; and (2) a myriad of other factors, some aiding and others opposing the true learning process; e.g., transfer from prior learning which plays a significant role in the learning process[14]. In practice, however, the actual subdivision of

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[12] Hirschmann, W. B., op. cit., p. 136.

[13] c. f.: Conway, R. W., and Schultz, A., Jr., op. cit., p. 42; and Levy, F. K., op. cit., p. 7 ff.

[14] Kolensky, W. R., The Application of a Manufacturing Progress Function to a High Volume Product with Missing Historical Data, Department of Industrial Engineering, Lehigh University, Bethlehem, Pa., May, 1964, (Master's Thesis), p. 6.

operator learning is difficult, if not impossible, to realize. One writer states that:

> -- the course of learning (is) determined by many factors, some of them inseparable from the learning process itself[15].

Among the factors which are beyond the true learning process[16] is a factor which is, in reality, induced learning introduced by the operator. One article states that:

> However, the operator may contribute improvements in task methods in some environments[17].

Hence, the subdivision of factors influencing operator learning becomes even more difficult to effect in practice since many of these improvement factors may go unnoticed other than by an experienced and determined observer.

Determinants of operator learning might include such factors as the employee's general experience, related experience on a specific job, education, job training, etc., when individual learning is involved. In group work where the productivity of each worker is a determining factor of the output, a weighted index of each of the attributes could be used as a possible way of measuring the effects of the attributes on operator learning.

Again, it is superfluous to the purpose of this thesis to pursue this line of development at this point other than to assert that the attributes affecting operator learning, e.g., job training,

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[15]Shephard, A. H., and Lewis, D., Prior Learning as a Factor in Shaping Performance Curves, U. S. Navy Technical Report SDC 938-1-4, State University of Iowa, Iowa City, Iowa, July 11, 1950, p. 2.

[16]The true learning process is meant to be the learning process in the limited sense of learning.

[17]Conway, R. W., andSchultz, A., Jr., op. cit., p. 42.

will generally be related to the rate of progress $\underline{\mu}$ and the initial efficiency $\underline{c}$ in the same manner as pre-production planning, i.e., related inversely to $\underline{\mu}$ and directly to $\underline{c}$.

PROCEDURE

Before proceeding with an empirical study of the predictive utility of the exponential learning function, it was desirable to bound the study to an extent that it became manageable but yet remained realistic. The desire for manageability of the study is well understood when considering the many facets of the factors involved in the industrial learning curve concept. However, the need to achieve realism in any study usually results in two opposing and somewhat contradictory goals. Realism to the theorist may be the inclusion in the study of all possible factors which enter into the consideration of the phenomenon of interest; whereas, to the practitioner realism may be the inclusion in the study of only as many of those factors which are necessary to provide results significant to his needs. Thus, the desire for realism becomes mainly a matter of a point of view. Therefore, realism is interpreted here more from the viewpoint of the practitioner; i.e., the inclusion of only those factors which are necessary to provide significant results.

From the standpoint of manageability, the empirical study was confined to the area of operator learning as defined in the previous section. Thus, individual operator performance data was used as the empirical means of testing the predictive utility of the exponential learning function. The most readily available information which portrayed operator performance was the operator percentage efficiency figures based upon standard work rates. This measure of performance had the added advantage of automatically compensating for minor changes[1] in methods, jigs, and fixtures, i.e., induced learning of a minor nature, by a corresponding change in the base rates from which the measure is determined.

The operator efficiency data was obtained for employees who had been newly hired or transferred to electronic component assembly lines[2]. This limitation of employing data for new hires or transferees not only gave information concerning the initial learning phase of the operators but was necessary to provide an accurate accounting of the cumulative production count of the units "produced" by the operator.

The data used in the empirical study was obtained from two manufacturing facilities of the Western Electric Company, Incorporated. For the usual proprietary reasons, the locations of the plants, the exact descriptions of the products, the identification of the operators,

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[1]It was assumed that major changes in methods and tooling were non-existent during the period of interest. However, major changes are usually easily identified through such documentation as cost reduction case records, etc. Conceivably, the maximum rate of output $\underline{Y}$ could increase significantly in such cases. Thus, a more general form of (19) would be:

$$y(x) = Y_0(1 + r_1 + r_2 + \text{-----} + r_n)\left[1 - e^{-(c + \mu x)}\right], \tag{23}$$

where $\underline{r}_1$, $\underline{r}_2$, ---, and $\underline{r}_n$ represent zero or the necessary ratios such that:

$$r_n = \begin{cases} 0 & \text{prior to change } n \\ \dfrac{Y_n - Y_{n-1}}{Y_0} & \text{following change } n \end{cases}, \tag{24}$$

where $\underline{Y}_0$ represents the original maximum rate of output and $\underline{Y}_n$ represents the maximum rate of output following change number $\underline{n}$ of a major nature.

[2]Assembly line, as used here, should not imply the true progressive assembly line; i.e., there is no "key" operator and the balance of the line is not critical to the operation of the line in that intermediate storage facilities and additional time on some operations are employed to compensate for unbalance when and where it occurs. In fact, some of the operations are performed on an entire lot at one time.

and the time periods during which the data was compiled must be withheld. However, these restrictions of disclosure do not occlude the validity of the study in any manner.

Having established the area to be considered in the study, the factors which were to be included within the context of the study had to be determined from the standpoint of realism; i.e., those factors which influence the rate of progress $\underline{\mu}$ and the initial efficiency $\underline{c}$ to a significant degree had to be ascertained. At this point, an assumption concerning the character of the operators was made which is obviously not strictly correct: all operators were assumed to be homogeneous prior to the time the operators were hired; i.e., such factors as age, sex, education and prior experience were not considered as factors which influence learning. This assumption may not be quite as unrealistic as it first appears since most of the operators were female high school graduates in their late teens and early twenties with no prior related experience. In fact, most operators had no manufacturing experience whatsoever; those who had such experience had gained it through employment in such unrelated manufacturing fields as hosiery knitting, etc.

Kilbridge[3], in discussing variables in group learning[4], has identified seven factors which influence learning:

(1) Task length (cycle time).

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[3]Kilbridge, M., op. cit., p. 517.

[4]Group learning, as discussed by Kilbridge, involves a progressive assembly line with a "key" operator; all operators on the line must progress at the same rate as this key operator.

(2) Group size.

(3) General level of skill and experience of the group.

(4) Complexity of the work.

(5) The degree of change the work presents from previous work.

(6) Worker motivation (e.g., wage incentives).

(7) Extraneous influences (e.g., supervisory pressure and worker-initiated restrictions).

It was postulated that the essence of the factors identified by Kilbridge is comprised by the following attributes which were used as factors influencing learning in the empirical study of the predictive utility of the exponential learning function:

(1) Cycle time.

(2) Related experience.

(3) The ratio of manual to machine effort.

(4) Diversification of job effort by the operator.

(5) Labor grade of job to which the operator is assigned.

(6) Work shift to which the operator is assigned.

The method used to test the predictive utility of the exponential learning function consisted basically of a comparison process of the results of correlation when fitting first the conventional industrial learning function, i.e., equation (7), and then the exponential learning function, i.e., equation (19), to the empirical data for each operator by a least-squares technique. As indicated earlier, (7) becomes linear through a logarithmic transformation, i.e.,

$$\ln y_i = \ln a + b \ln x_i \tag{25}$$

Similarly, (19) may be transformed into a linear function, e.g.,

$$\ln\left(\frac{Y}{Y-y_i}\right) = c + \mu x_i . \tag{26}$$

In actuality, (25) and (26) were fitted to the paired values $(\underline{x}_i, \underline{y}_i)$ of the empirical data for each operator by the standard least-squares process for linear functions.[5] Computations were facilitated by means of an IBM Data Processor using FORTRAN compiled programs.

Since no postulations or assumptions were made concerning the error distribution of the efficiencies $\underline{y}$ other than that the mean and variance of the error distribution were essentially constant over the period of time under consideration,[6] no sophisticated statistical analyses were made of the correlations. The coefficient of correlation $\underline{R}$ was considered as being sufficiently indicative of the nature of the fit obtained by the least-squares technique in each case. Thus, for each operator, two coefficient of correlation values were available

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[5] c.f.: Dixon, W. J., and Massey, F. J., Jr., Introduction to Statistical Analysis, McGraw-Hill Book Co., Inc., New York, 1957, p. 189 ff.

[6] The measured cumulative production $\underline{x}$ was considered to be an independent variable with no significant measurement error.

for testing by some non-parametric statistical means in evaluating the composite result of the correlation procedure.

Having ascertained the values of $\underline{c}$ and $\underline{\mu}$ for each operator by means of fitting (26) to the paired values ($\underline{x}_i$, $\underline{y}_i$) of the data for that operator, the effect of those variables $\underline{z}_i$, which were postulated earlier as influencing $\underline{c}$ and $\underline{\mu}$ by the relationships expressed by (18) and (21), respectively, were tested to determine if these variables did indeed influence those two parameters. The method used to accomplish this testing consisted basically of an evaluation of the coefficients of multiple correlation obtained when fitting (18) and (21) to the appropriate operator data for all operators of each assembly line.

From (18), $\underline{\mu}$ for each operator may be stated as:

$$\mu_j = \alpha_o + \sum_{i=1}^{n} \alpha_i z_{ji}, \qquad j = 1, 2, \ldots, m, \qquad (27)$$

where $\underline{n}$ is the number of operator attributes $\underline{z}_{ji}$ under consideration and $\underline{m}$ is the number of operators being considered from a particular assembly line. Here, $\underline{\alpha}_o$ and the $\underline{\alpha}_i$'s are the regression coefficients of a multiple correlation process for the $\underline{m}$ expressions for the $\underline{\mu}_j$'s. Likewise, from (21), $\underline{c}$ for each operator may be stated as:

$$c_j = \beta_o \prod_{i=1}^{n} z_{ji}^{\beta_i}, \qquad j = 1, 2, \ldots, m, \qquad (28)$$

where the $\underline{z}_{ji}$'s have the same definition as used in (27) and $\underline{\beta}_o$ and the $\underline{\beta}_i$'s are the regression coefficients of a multiple correlation process for the $\underline{m}$ expressions for the $\underline{c}_j$'s. In actuality, the logrithmic transformation of (28) was used in the multiple correlation procedure for determining the β's, i.e.,

$$\ln c_j = \ln \beta_o + \sum_{i=1}^{n} \beta_i \ln z_{ji}, \; j = 1, 2, \ldots, m, \qquad (29)$$

was used instead of (28) for correlation purposes. Computations for the multiple correlations were facilitated by the use of an IBM 1620 Data Processor using a library program[7] written for that purpose. Through the multiple correlation procedure using (27) and (29) with the values of the μ_j's and c_j's, respectively, obtained earlier and the appropriate operator attributes z_{ij} for each case, general expressions for the initial efficiency c and the rate of progress μ were derived for each assembly line. The coefficients of multiple correlation for each expression were available for the evaluation of the fits. Again, the coefficient of multiple correlation R was considered sufficiently indicative of the nature of the fit obtained by the multiple correlation procedure in each case. No further statistical testing was deemed advisable in this case, the reason for which will become evident in the next section.

[7] c.f., Leeson, D. N., Multiple Linear and Non-Linear Regression Analysis for the IBM 1620, (undated, unnumbered SHARE document).

RESULTS AND ANALYSIS

The paried values (x_i , y_i) of the raw operator efficiency and cumulative production data which were used in the correlation procedures are given in Tables 1 through 18. Each table also shows the resulting least-squares equations for the exponential and the conventional learning curves fitted to the data, along with the corresponding coefficients of correlation. For comparison purposes, the efficiency values obtained with each value of x_i substituted into the two least-squares equations are shown alongside the raw data.

Table 19 summarizes the information concerning the coefficients of correlation obtained in fitting each of the curves to the data for each of the 18 operators. Also shown in that table are the differences of these two coefficients of correlation for each of the operators. Assuming that the differences between the coefficients of correlation for each of the operators are independent, i.e., that the outcome of one pair of coefficients of correlation is in no way influenced by the outcome of any other pair, the non-parametric sign test may be applied to the differences of the coefficients. In this case, the null hypothesis is that each difference has a probability distribution with median equal to zero.[1] In essence, this hypothesis postulates that fitting the conventional learning function to the operator data will provide no significantly better, or worse, coefficient of correlation than that obtained when fitting the exponential learning curve.

[1]The probability distribution for each difference need not be the same for all differences; i.e., the only commonality of the probability distributions assumed is the zero median.

Using the same designations as employed by Dixon and Massey[2], the number of observations $\underline{N}$ is 18 and the number of times the less frequent sign occurs $\underline{r}$ is 3. Entering a table of critical values of $\underline{r}$ for the sign test[3] with $\underline{N}$ equal to 18, the value of $\underline{r}$, i.e., 3, is significant at the 1% level; therefore, the null hypothesis must be rejected at this level. Thus, in this case, the hypothesis that the exponential learning function provides a fit to the operator data as well as the conventional learning function must be rejected.

The problem area associated with the relatively poor correlation results of the exponential learning function may be seen graphically by plotting the operator paired data ($\underline{x}_i$, $\underline{y}_i$) along with the two resulting curves of the correlation analysis for each case on Cartesian coordinates. Figure 5 illustrates a typical case where the exponential function resulted in a poor fit to the operator data (operator number 3) relative to the fit of the conventional function to the same data. The exponential function fails to accurately portray the early phase of operator learning during which the rate of change of efficiency is relatively large in comparison with that of the later phases of the learning process for individual operators.

Figure 6 illustrates one of the two cases in which the fits of the two functions to the operator data (operator number 10) are comparable with respect to the coefficients of correlation. In each of these cases, the exponential function provided a questionably better

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[2]Dixon, W. J., and Massey, F. J., Jr., op. cit., p. 281.

[3]c.f.: Dixon, W. J., and Massey, F. J., Jr., op. cit., p. 417.

FIGURE 5. ILLUSTRATION OF LEAST-SQUARES FITS OF LEARNING FUNCTIONS TO OPERATOR DATA (OPR. NO. 3)

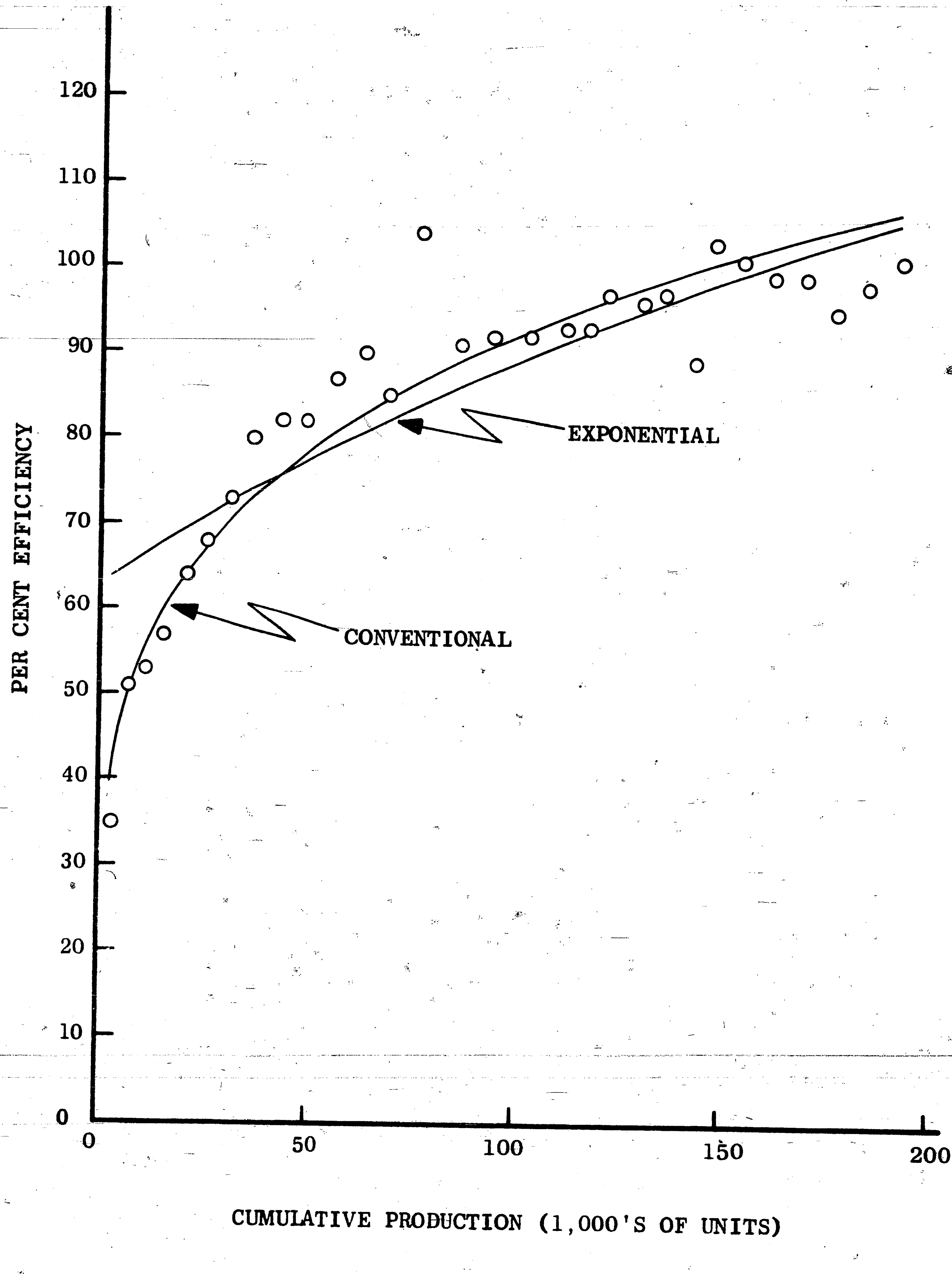
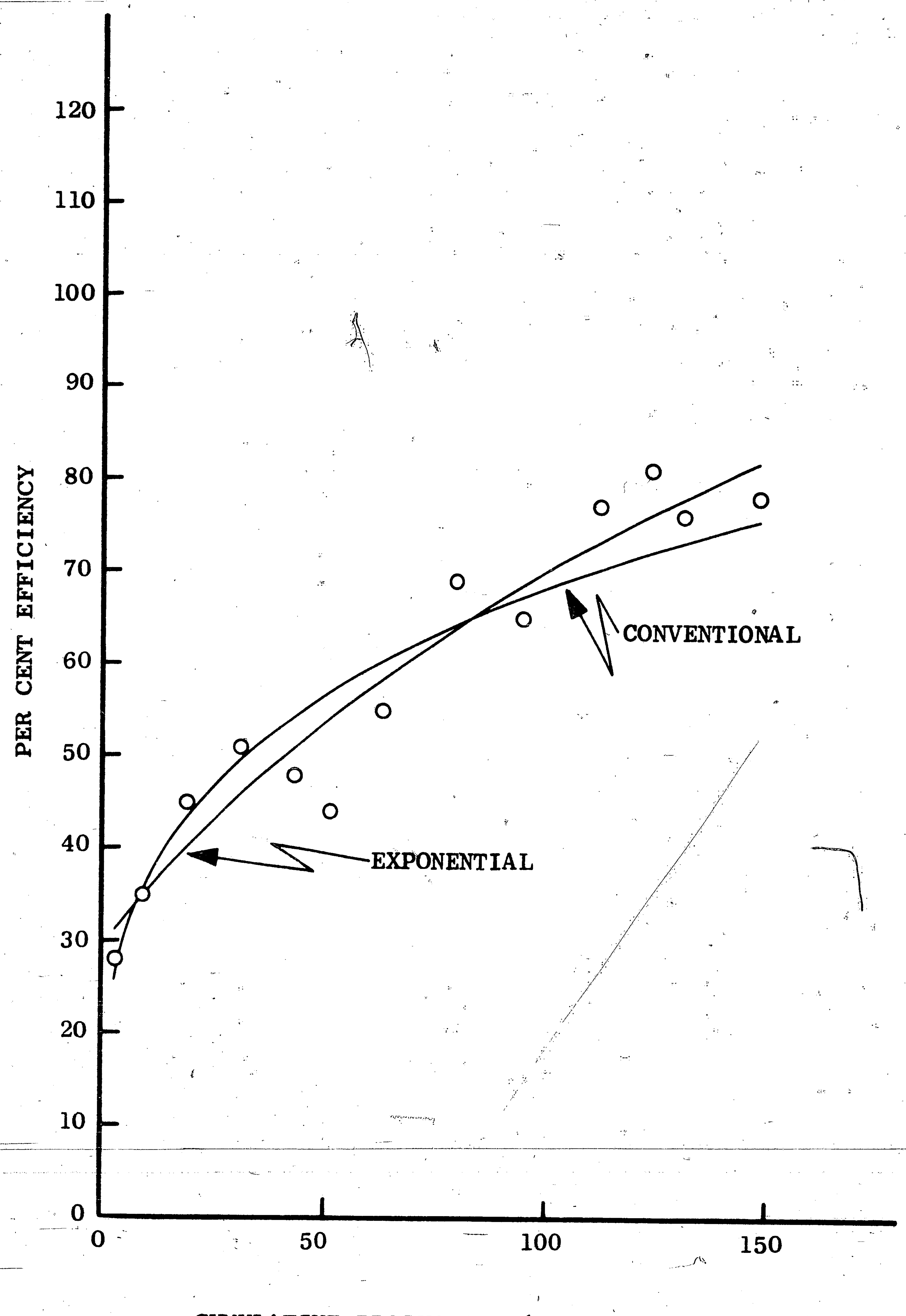


FIGURE 6. ILLUSTRATION OF LEAST-SQUARES FITS
OF LEARNING FUNCTIONS TO OPERATOR DATA (OPR. NO. 10)

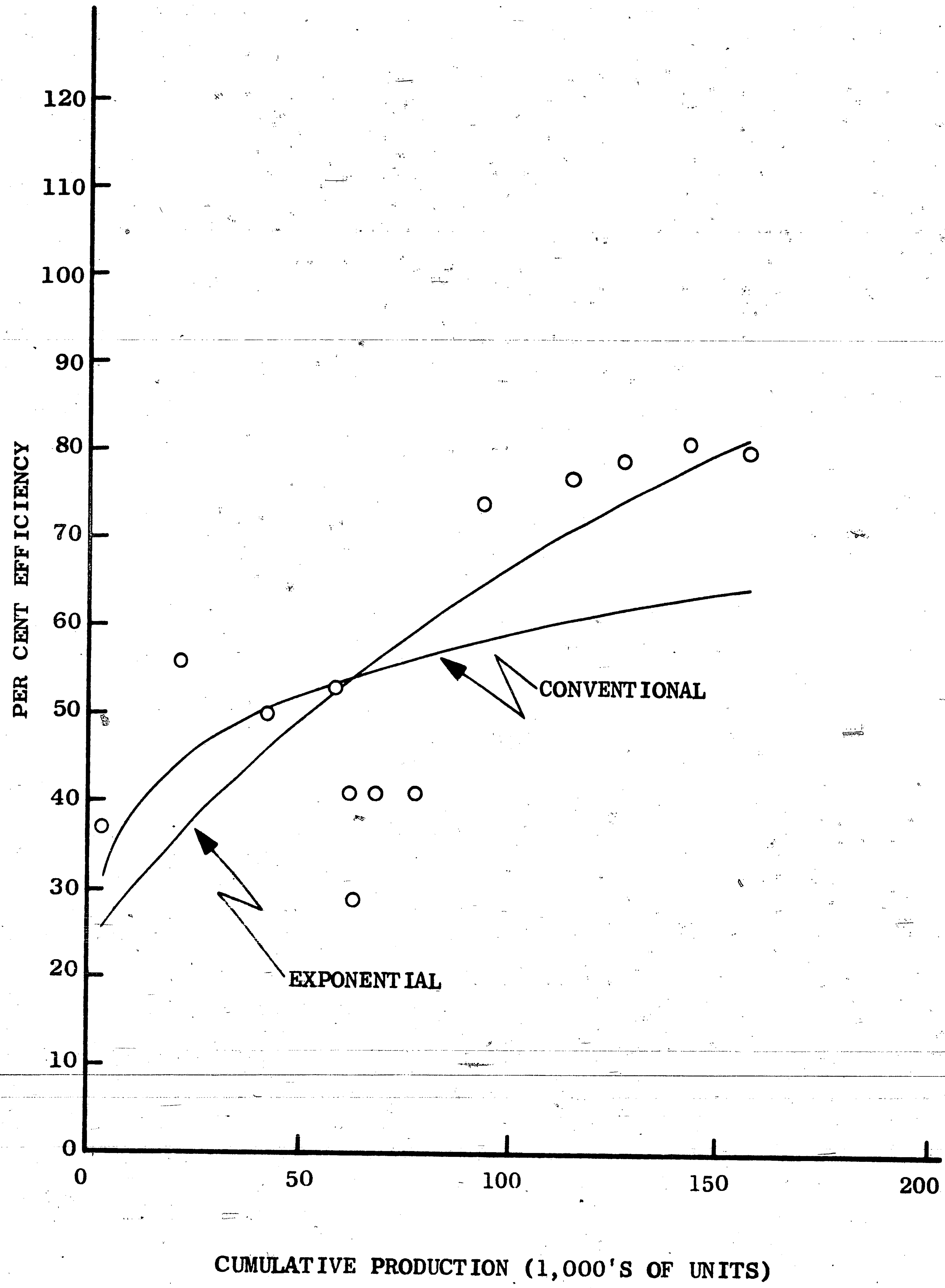


fit that that of the conventional function. As may be seen by observation of Figure 6, the operator data is highly variable relative to both functions. In fact, the data tends to portray three separate and distinct phases of "accelerated" learning typical of the early phase of the learning process. However, each phase of "accelerated" learning is followed by a period of lower efficiency. No assignable causes for these lower efficiencies, such as significant changes in work assignments, were observed other than that the operator was absent from work for a period of one week between the eleventh and twelfth observations.

Figure 7 illustrates the case in which the exponential function provided a markedly better fit to the operator data (operator number 18) than did the conventional function with respect to the coefficients of correlation. However, upon observation, neither the exponential nor the conventional learning function accurately portrays the true situation exhibited by the data because of its highly erratic nature. It was questionable whether this case should have been included in the study because of the lack of any conclusive results which might have been drawn from it. However, no criteria was established at the beginning of the study to limit the cases investigated other than those mentioned earlier; therefore, this case was included in the study ipso facto.

Figure 5 typifies the graphic representation of the remaining fourteen cases investigated. A cursory review of Tables 1 through 18 will show that the undesirable characteristic of the exponential learning function which was discussed earlier, i.e., the characteristic of being incapable of portraying the initial phase of the

FIGURE 7. ILLUSTRATION OF LEAST-SQUARES FITS OF LEARNING FUNCTIONS TO OPERATOR DATA (OPR. NO. 18)



learning process, is present to some degree in each of these fourteen cases.

Despite the evidence that the exponential learning curve provided a less exacting fit to the empirical data than does the conventional learning curve, it was decided that the study of the predictive utility of the exponential curve should be continued in pursuit of the factors affecting the progress rate $\underline{\mu}$ and initial efficiency $\underline{c}$ of each worker in connection with the exponential learning function. These factors were postulated in a previous section; it was the intent of this portion of the study to determine whether these factors did indeed influence $\underline{\mu}$ and $\underline{c}$.

Before proceeding with the discussion concerning the study of the factors which were postulated as affecting $\underline{\mu}$ and $\underline{c}$, it should be noted that operators number 1 through 6 performed only one specified series of task elements repeatedly out of a possibility of five such series as is shown in Table 20. These series of task elements consisted of certain procedures in the mounting and adjusting of elemental parts of an electronic component using hand tools and small capacitor-discharge welding equipment. On the other hand, operators number 7 through 18 performed from one to nine task elements each out of a possibility of 19 such task elements as is shown in Table 21. These series of task elements constituted a portion of the manufacturing procedure for a second electronic component which was manufactured at a separate location from that at which the first component was manufactured. The operators at the second location processed the units through a task element in lots varying in size from tens of units in sample lots to thousands of units in production lots. The cumulative production for each operator at this manufacturing location was measured by the number of units processed through any task element performed by that operator since the operator did not necessarily process the same number of units consecutively through all task elements performed by that operator as indicated by Table 21.

The six operators at the first manufacturing location had no related manufacturing experience; had one task assignment, each of which was of the same labor grade; and were assigned to the same work shift. Thus, the only variability among the operators under the conditions of the postulations with the assumption of homogeneity of the operators at the time of initial employment was in the cycle time and the ratio of manual to machine effort of each operator's work assignment. These factors are shown in Table 20 as standard base credit hours per 100 units produced and per cent manual effort, respectively.

For the purpose of correlation, μ for each operator at the first manufacturing location may be stated as:

$$\mu_j = \alpha_0 + \alpha_1 z_{j1} + \alpha_2 z_{j2}, \qquad j = 1, 2, \ldots, 6, \qquad (30)$$

where

z_{j1} = base hours per 100 units for the task performed by operator j,

z_{j2} = per cent manual effort for that task,

and α_0, α_1, and α_2 are the regression coefficients of the multiple correlation of the six expressions for the μ_j's. The multiple correlation procedure provided a general expression of the μ_j's for the operators at that manufacturing location which was of the form of

$$\mu_j = (1.55 + 10.08\ z_{j1} - 5.89\ z_{j2}) \times 10^{-6} \tag{31}$$

with a coefficient of multiple correlation of 0.912.

Similarly, c for each operator at the first location may be stated as:

$$\ln c_j = \ln \beta_0 + \beta_1 \ln z_{j1} + \beta_2 \ln z_{j2}, \quad j = 1, 2, \ldots, 6, \tag{32}$$

where z_{j1} and z_{j2} have the same definition as used in (30) and β_0, β_1 and β_2 are the regression coefficients of the multiple correlation of the six expressions for the c_j's. The multiple correlation procedure provided a general expression for the c_j's for the operators at that location which was of the form of

$$c_j = 0.698\ z_{j1}^{-0.377}\ z_{j2}^{0.575} \tag{33}$$

with a coefficient of multiple correlation of 0.862.

The twelve operators at the second manufacturing location had no related manufacturing experience and had task assignments of the same labor grade. However, the operators at the second location had multiple job assignments and operators from more than one work shift were included in the study. Thus, in addition to the two factors which were considered in the study of the factors affecting learning in the case of the operators at the first location, diversification of job effort and work shift could be included in the consideration of factors affecting learning of the operators at the second manufacturing location. These factors were represented by the number of task assignments and the number of the work shift, i.e., 1 for first shift (normal working hours), etc., of each operator.

A problem was encountered in representing the base hours per 100 units (cycle time) for the operators who were assigned more than one task. As discussed earlier in this section, the operators who were assigned multiple tasks did not necessarily process each product lot through all tasks; thus, the cumulative cycle time of all tasks assigned to the operator was considered inappropriate. The arithmetic mean was likewise considered inappropriate. The only true representation of cycle time would have been a weighted average of the individual times based upon the number of product units processed through each task. However, this weighted average technique would have destroyed the generality of the model for future use. The problem was resolved by using an adaptation of the coefficient of variation.

Since it was desirable that the resulting estimate of cycle time yield the original base hours per one hundred units for one task assignment, the sum of the arithmetic mean $\bar{z}_1$ and the ratio of the variance s_z^2 to $\bar{z}_1$ was used as a first measure of this estimate; i.e., the location parameter was modified by a factor expressing the degree of spread in terms of that location parameter, or that

$$\hat{z}_1 = \bar{z}_1 + \frac{s_z^2}{\bar{z}_1} , \tag{34}$$

where $\hat{z}_1$ is the first measure of the cycle time of the several tasks assigned to one operator. This measure provided the desirable characteristic of yielding the base hours per 100 units for one task assignment. It was also postulated that this measure of cycle time would be further modified by the number of separate tasks assigned to the operator, or that the final estimate of the factor associated with cycle time is

$$\hat{z}_1' = \hat{z}_1 z_3^p = z_3^p (\bar{z}_1 + \frac{s_z^2}{\bar{z}_1}), \tag{35}$$

where

$\hat{z}_1'$ = the estimate of cycle time for multiple task assignments,

$\bar{z}_1$ = the arithmetic mean of the cycle times of the several tasks,

s_z^2 = the variance of these cycle times.

and

z_3^p = the number of these tasks raised to the power p, a value to be determined later.

Of a similar nature was the determination of the per cent of manual effort for multiple task assignments. However, in this case a weighted average of the per cent of manual effort of the individual tasks weighted by the corresponding individual cycle times was used as an estimate of z_2 without loss of generality, although it is conceded that this estimate is not without its shortcomings.

For purposes of correlation, μ for each operator at the second manufacturing location may be stated as:

$$\mu_j = \alpha_0 + \alpha_1 \hat{z}_{j1} z_{j3}^p + \alpha_2 \hat{z}_{j2} + \alpha_3 z_{j3} + \alpha_4 z_{j4}, \quad j = 1, 2, \ldots, 12, \tag{36}$$

where

$\hat{z}_{j1}$ = a first measure of the estimate of the cycle time (base hours per 100 units) and equals $\bar{z}_1 + \frac{s_z^2}{\bar{z}_1}$ for the tasks performed by operator j,

$\hat{z}_{j2}$ = an estimate of the per cent of manual effort for those tasks and equals the average of the individual tasks' per centages weighted by the corresponding individual cycle times,

z_{j3} = the number of tasks performed by operator j

z_{j4} = the work shift, i.e., 1, 2, or 3, to which the operator is assigned,

p = an exponent of z_{j3} to be determined,

and α_0, α_1, α_2, α_3, and α_4 are the regression coefficients of the multiple correlation of the twelve expressions for the μ_j's. The multiple correlation procedure provided a general expression for the μ_j's for the operators at that location which was of the form of

$$\mu_j = (72.95 - 0.0001408\ \hat{z}_{j1}\ z_{j3}^{6} - 71.04\ \hat{z}_{j2} + 0.3031\ z_{j3} + 0.02432\ z_{j4}) \times 10^{-5} \qquad (37)$$

with a coefficient of multiple correlation of 0.703. In this case, p was determined through a trial search procedure for the largest coefficient of multiple correlation.

Similarly, c for each operator at the second location may be stated as:

$$\ln c_j = \ln \beta_0 + \beta_1 \ln \hat{z}_{j1}\ z_{j3}^{6} + \beta_2 \ln \hat{z}_{j2} + \beta_3 \ln z_{j3} + \beta_4 \ln z_{j4},$$
$$j = 1, 2, \ldots, 12 \qquad (38)$$

where $\hat{z}_{j1}$, $\hat{z}_{j2}$, z_{j3}, and z_{j4} have the same definition as used in (36) and β_0, β_1, β_2, β_3, and β_4 are the regression coefficients of the multiple correlation of the twelve expressions for the c_j's. The multiple correlation procedure provided a general expression for the c_j's for the operators at that locations which was of the form of

$$c_j = 0.307\ \hat{z}_{j1}^{\,0.354}\ \hat{z}_{j2}^{\,1.15}\ z_{j3}^{\,0.078}\ z_{j4}^{\,0.382} \qquad (39)$$

with a coefficient of multiple correlation of 0.737[4].

From equations (31) and/or (37), it may be concluded that:

1. The per centage of manual effort (a relative measure in some degree of the complexity) of the task assigned to an operator

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[4] Equation (39) has the two z_{j3} terms combined as opposed to separate terms as suggested by (38).

affects the rate of progress μ in the direction expected; i.e., the higher the percent of manual effort involved in a task, the lower the value of μ associated with that task.

2. The cycle time of the task affects μ in the case of the second manufacturing location in the direction expected; i.e., the longer the cycle time, the lower the value of μ. However, in the first case, cycle time has the opposite effect. Kilbridge has stated that:

> When the task length is very short a simple and restricted motion pattern is repeated continuously. Observations indicate that this induces cramping and excessive muscle fatigue which inhibit the worker's ability to maintain a uniformly fast pace. The reaction is known in industry and is commonly called "short-task" fatigue.[5]

He proceeds to produce empirical evidence to show that the ultimate pace attainable for an operator in a manufacturing process is a function of task length (cycle time) and that this attainable pace is highest for tasks of from one-third minutes to about one and one-half minutes. The three jobs investigated at the first location were of 0.6, 0.3, and 0.24 minutes each, approximately, while the effective cycle times, for the most part, were of at least a magnitude greater at the second location. It is suggested that the same "short-task" phenomenon is present in the cycle times' effect upon μ at the first location..

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[5] Kilbridge, M., op. cit., p. 519.

3. The number of task assignments affects $\underline{\mu}$ in the direction expected; i.e., the greater the number of tasks assigned to an operator, the lower the value of $\underline{\mu}$. The sign of $\underline{\alpha}_3$ in equation (37) is misleading in that it indicates the opposite effect; however, the number of tasks to the sixth power in the term involving the effective cycle time masks the effect of the term involving only the number of tasks.
4. The shift to which the operator is assigned affects $\underline{\mu}$ in the direction such that the higher the shift number, with 1 representing normal working hours, etc., the higher the value of $\underline{\mu}$. It is suggested that, since only the first and the third shifts were involved, the third shift operators progressed at a higher rate of learning because the experimental and engineering evaluation product lots which were processed through the assembly line during the first shift reduced the learning rate of those operators through an interference effect.

No conclusions are drawn from equations (33) and (39) concerning the factors which were postulated as affecting the initial efficiency. It is contended that these equations are invalid for such purposes in these two cases in that, due to the poor fit of the exponential curve to the early phase of the learning process, the initial efficiencies obtained are entirely dependent upon the rate of progress. This contention is substantiated by the nature of the regression coefficients obtained in the general expressions for $\underline{c}$. Almost without exception, these coefficients indicate that the factors serve to affect $\underline{c}$ in the opposite direction to the manner in which they affected $\underline{\mu}$. A cursory reference to Tables 1 through 18 and the corresponding illustrations will show that any increase in slopes of the curves, i.e., the effect of $\underline{\mu}$, will essentially decrease the resulting initial efficiencies, and vice-versa with respect to a decrease in slopes, when poor fits to the early portion of the learning curves are obtained as in the two cases under discussion.

In summary, the intent of the exponential learning function was two fold:

1. A more realistic portrayal of the cost-quantity relationship found in almost all production processes.
2. A portrayal of the quantitative cause and effect relationships of the initial productivity and the rate of progress on a production process with those factors which influence these parameters.

With regard to each of these areas, this thesis has shown the following:

1. To determine the predictive utility of the exponential learning function in comparison with that of the conventional learning function, data for eighteen operators of two separate manufacturing operations was obtained for an empirical study. Of the eighteen cases, the exponential function provided results which represented an improvement over the conventional learning curve in only three cases. In two of these cases, the amount of improvement was relatively small. Regardless of the degree to which these three cases represented improve-

ments, these cases were significant at the 1% level in the non-parametric sign test applied to the differences of the coefficients of correlation for the two learning curves when fitted to the individual operator data. The result of this test indicates rejection of the hypothesis that the exponential function provides a fit to the operator data as well as does the conventional curve.

The major problem area associated with the poor correlation results of the exponential learning function was identified as the inability of the function to accurately portray the early portions of the operators' efficiency curves (the initial learning phase), during which the rates of change in the efficiencies were relatively large in comparison with the later portions of the curves.

2. In ascertaining the ability of extensions of the exponential learning function to portray the quantative cause and effect relationships of the initial efficiency and the rate of efficiency progress to factors which were postulated as affecting these parameters, the results of that portion of the correlation study discussed above were used notwithstanding the poor fits to the data obtained. Hence, the degree to which that ability may be expressed must be made with qualifications. For this reason, the credulity of the outcome of the relationships of the factors postulated as affecting initial efficiency was rejected in toto because of the extremely poor indication of initial efficiency by the exponential

function. The fit of the function to later portions of the data was more reasonable; thus, the rates of efficiency progress were accepted from the correlation study without modification. The results of the investigation of the factors which were postulated as influencing the progress rates indicated that these factors affected the rates in the directions expected. The degree to which the factors and the rates of progress were correlated was not subjected to statistical testing; however, the coefficients of correlation were within reasonable bounds when qualified by the fits obtained in the earlier correlation study.

CONCLUSIONS

1. The exponential learning function of the form $y = Y\left[1-e^{-(c+\mu x)}\right]$ does not represent a general expression describing the cost-quantity relationship for that category of learning attributable to the individual operator in a manufacturing process. The exponential function lacks the ability to describe the early phase of operator learning in those manufacturing situations in which neither of the following conditions exists:
   a. An unusually high content of machine effort is involved in the execution of the task assigned to the operator.
   b. The operator performing the task is relatively well grounded in the execution of the task through experience in the performance of tasks which provide positive transfer of learning.

   These two characteristics preclude the existence of a high degree of progress typical of the early phase of the learning process associated with the manufacturing operations studied in this thesis. If either or both of the above conditions exist, it is conceivable that the exponential function could be applied with success.
2. During the early phase of the learning process, the conventional learning function of the form $y = ax^b$ more accurately portrays the general cost-quantity relationship associated with the individual operator. This early phase of the learning process is generally characterized by a high rate of change in the progress in comparison with that of the later phases. Beyond this initial phase, it is also conceivable that the exponential function would provide an

acceptable portrayal of the remaining portion of the learning process.

3. The manner in which the factors which were postulated in this thesis as influencing individual operator learning affect the rate of progress was as expected; i.e.:

   a. The higher the per centage of manual effort, the lower the progress rate.

   b. The higher the cycle time within certain limits, the lower the progress rate. The effect of cycle time upon the progress rate is reversed when cycle times are less than approximately one-half of a minute.

   c. The higher the number of tasks assigned to one operator, the lower the progress rate.

   d. The more interference effects introduced into the manufacturing process, the lower the progress rate. Interference effects are defined to include such departures from normal operations as product lots processed for engineering evaluation and experimental purposes which may require additional operations or special manipulations due to the nature of the studies to which the lots are subjected.

4. Finally, a blanket dismissal of the exponential learning function may not be made as a result of the present study. However, if the exponential function is to be employed, the application of the function must be undertaken advisedly, especially realizing that it is by no means general in nature.

AREAS FOR FURTHER STUDY

The predictive utility of the exponential learning function was investigated with respect to only one of the categories of learning. Further research and documentation regarding the type of activities to which the exponential function could be successfully applied within this category represent opportunities for further contributions to the industrial learning curve literature. Particularly, the situation in which it is desirable to apply a learning curve to an operator who has passed the initial phase of the learning process and for whom no historical data exists represents only one area to be investigated. Additional areas for further study within the same learning category are the cases in which the operator begins the performance of a task for which he has considerable experience and/or the task being performed contains a high content of machine effort.

Studies of the predictive utility of the exponential learning function with respect to the entire category of induced learning represent opportunities for major contributions fo the field. The variations of approach to this area are many, with the subject of engineering effectiveness being one of the prime areas for investigation. Further investigation of the application of the function to the total learning concept in which the categories of operator and induced learning are brought together with the resulting interplay of the two could provide useful information for possible employment of the function in the industrial environment of a manufacturing process. Multiproduct shops would provide variations within this total field.

Finally, it should be stressed that the exponential learning function does not eliminate the total problem of convexities which tend to develop in empirically derived progress curves; i.e., the convexity problem has been approached at only one level but the problem of obtaining a function of the same form when adding two such functions has not been solved. Thus, it would be desirable that some function be developed which is general for all levels of industry from the categories of the learning process within one operation, to the departmental level, to the total manufacturing program for a major project, and finally to the entire industry. This all encompassing function would necessarily have to be based upon theoretical considerations and be capable of showing those factors which influence the learning process at each level.

TABLE 1

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 1

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 140\left[1 - e^{-(.851+.971\times10^{-6}x_i)}\right]$	0.782
Conventional	$y_2 = 27.385x_i^{.100}$	0.895

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES	CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
5,403	53	80.50	64.44
10,503	67	80.80	68.88
20,153	80	81.35	73.52
29,103	75	81.86	76.27
40,554	84	82.50	78.85
51,991	85	83.14	80.83
59,091	88	83.53	81.88
67,591	86	83.99	82.98
75,842	87	84.44	83.95
84,942	89	84.93	84.90
93,692	87	85.39	85.74
102,692	88	85.87	86.53
112,042	91	86.36	87.29
119,160	91	86.73	87.83
128,374	90	87,20	88.49
135,674	91	87.58	88.98
145,252	95	88.06	89.59
154,502	90	88.53	90.14
162,702	92	88.93	90.61
171,226	88	89.35	91.08
180,643	94	89.82	91.57
192,793	100	90.40	92.16
203,693	92	91.02	92.76
215,693	90	91.50	93.21
224,543	89	91.91	93.58
230,043	91	92.17	93.81
241,893	93	92.71	94.28
251,814	97	93.17	94.66
263,864	91	93.71	95.11
270,587	94	94.01	95.35
278,237	95	94.35	95.61
288,287	92	94.80	95.95
297,887	95	95.22	96.27

TABLE 1 (cont'd.)

CUMULATIVE PRODUCTION COUNT (x_i)	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
305,887	93	95.56	96.52
315,437	92	95.97	96.82
324,687	96	96.37	97.10
333,987	95	96.76	97.38
343,187	95	97.14	97.64
352,887	97	97.55	97.91
363,226	98	97.97	98.20
372,451	93	98.34	98.44
382,451	100	98.75	98.71

TABLE 2

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 2

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 140\left[1 - e^{-(.725+.503\times10^{-5}x_i)}\right]$	0.819
Conventional	$y_2 = 7.316x_i^{.230}$	0.930

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
928	31	72.51	35.27
2,065	32	72.89	42.40
3,358	35	73.33	47.43
5,226	50	73.95	52.52
7,352	57	74.65	56.82
9,708	65	75.42	60.57
12,482	73	76.32	64.19
14,848	82	77.07	66.80
17,696	79	77.96	69.56
20,890	86	78.95	72.27
24,104	86	79.93	74.69
27,402	90	80.92	76.93
30,588	90	81.86	78.91
33,215	91	82.62	80.42
36,610	91	83.59	82.24
39,744	86	84.47	83.81
43,958	95	85.64	85.78
47,263	96	86.53	87.23
50,102	98	87.29	88.41
52,539	95	87.93	89.38
56,035	94	88.84	90.72
59,913	104	89.83	92.13
63,625	90	90.76	93.41
66,168	94	91.38	94.26
68,868	90	92.04	95.13
72,508	99	92.91	96.27
76,236	106	93.78	97.38
80,050	104	94.66	98.49
83,889	102	95.53	99.56
87,570	102	96.34	100.54
91,400	102	97.17	101.54
95,235	105	97.99	102.51
99,065	101	98.79	103.44

TABLE 2 (cont'd.)

CUMULATIVE PRODUCTION COUNT (x_i)	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
102,705	100	99.54	104.31
105,855	105	100.18	105.03
109,729	103	100.94	105.91
112,669	100	101.52	106.56
116,422	100	102.24	107.36
120,018	99	102.91	108.12
123,799	99	103.61	108.89
127,651	101	104.31	109.66
131,451	100	104.99	110.41
135,301	102	105.66	111.15
139,184	102	106.32	111.87
143,089	103	106.98	112.59
145,809	102	107.42	113.08
149,694	102	108.05	113.76
153,627	103	108.68	114.45
157,531	104	109.29	115.11

TABLE 3

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 3

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 140\left[1 - e^{-(.600+.410\times10^{-5}x_i)}\right]$	0.847
Conventional	$y_2 = 6.959x_i^{.224}$	0.968

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
	ACTUAL (y_i)	CALCULATED EXPONENTIAL (y_1)	CALCULATED CONVENTIONAL (y_2)
2,390	35	63.93	39.71
6,230	51	65.12	49.23
10,503	53	66.42	55.36
15,050	57	67.78	60.01
20,268	64	69.30	64.16
25,708	68	70.86	67.67
31,642	73	72.52	70.90
36,644	80	73.89	73.28
42,858	82	75.55	75.90
49,422	82	77.26	78.37
56,473	87	79.04	80.75
63,479	90	80.77	82.89
68,777	85	82.04	84.40
77,122	104	83.99	86.60
86,872	91	86.18	88.94
94,292	92	87.79	90.59
103,202	92	89.66	92.45
112,194	93	91.48	94.20
117,362	93	92.50	95.15
122,024	97	93.39	95.99
130,350	96	94.96	97.42
135,530	97	95.90	98.28
143,340	89	97.29	99.52
147,940	103	98.09	100.23
154,290	101	99.16	101.18
162,040	99	100.44	102.30
169,940	99	101.70	103.40
177,690	95	102.89	104.44
185,390	98	104.04	105.44
193,190	101	105.17	106.41

TABLE 4

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 4

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 140\left[1 - e^{-(.629+.292\times10^{-5}x_i)}\right]$	0.647
Conventional	$y_2 = 6.554x_i^{.224}$	0.900

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
	ACTUAL (y_i)	CALCULATED EXPONENTIAL (y_1)	CALCULATED CONVENTIONAL (y_2)
1,687	22	65.70	34.80
5,098	43	66.44	44.59
9,193	64	67.31	50.90
13,890	60	68.30	55.84
19,398	70	69.45	60.18
26,487	74	70.89	64.54
32,621	76	72.12	67.63
38,686	78	73.31	70.26
44,132	72	74.36	72.37
48,752	79	75.24	74.00
55,318	84	76.47	76.13
61,220	76	77.56	77.88
65,510	92	78.33	79.08
72,444	92	79.57	80.88
78,449	80	80.62	82.34
85,233	87	81.78	83.88
94,435	88	83.33	85.84
100,778	82	84.37	87.10
107,082	82	85.38	88.29
114,202	81	86.50	89.57
119,370	98	87.31	90.47
124,032	108	88.02	91.25
132,358	95	89.27	92.59
137,538	92	90.03	93.39
145,348	91	91.15	94.55
149,948	98	91.81	95.22
155,108	89	92.53	95.94
161,334	83	93.38	96.79
168,004	91	94.28	97.68
174,919	100	95.19	98.56
181,453	91	96.04	99.38
188,259	92	96.91	100.20
195,369	93	97.79	101.04

TABLE 4 (cont'd.)

CUMULATIVE PRODUCTION COUNT (x_i)	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
202,483	95	98.66	101.85
209,903	95	99.54	102.68
217,536	95	100.44	103.50

TABLE 5

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 5

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 140\left[1 - e^{-(.728+.608\times10^{-5}x_i)}\right]$	0.921
Conventional	$y_2 = 13.599x_i^{.177}$	0.964

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
2,384	50	73.39	53.96
5,174	62	74.51	61.87
7,073	63	75.26	65.39
9,369	64	76.16	68.72
11,891	67	77.13	71.67
14,598	75	78.15	74.32
17,399	81	79.20	76.66
20,308	80	80.26	78.78
23,388	83	81.37	80.77
25,858	87	82.24	82.21
29,191	90	83.40	83.99
32,741	95	84.61	85.71
36,151	94	85.74	87.23
39,681	97	86.90	88.68
42,934	96	87.94	89.92
45,746	87	88.82	90.93
48,376	92	89.63	91.84
51,754	96	90.65	92.94
55,316	98	91.71	94.04
59,526	96	92.93	95.26
62,886	96	93.88	96.19
65,377	94	94.57	96.85
68,967	98	95.55	97.77
73,287	96	96.70	98.83
77,047	100	97.68	99.71
79,777	102	98.38	100.32
82,817	101	99.14	100.99
86,625	102	100.07	101.79
90,420	102	100.98	102.57
94,064	99	101.84	103.28
96,821	100	102.70	104.00
101,501	102	103.52	104.68
105,311	102	104.36	105.37

TABLE 5 (cont'd.)

CUMULATIVE PRODUCTION COUNT (x_i)	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
109,151	105	105.16	106.02
112,976	100	105.79	106.53
116,681	101	106.74	107.29
120,596	104	107.52	107.92

TABLE 6

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 6

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 140\left[1 - e^{-(.603+.791\times10^{-5}x_i)}\right]$	0.948
Conventional	$y_2 = 9.393x_i^{.210}$	0.969

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
	ACTUAL (y_i)	CALCULATED EXPONENTIAL (y_1)	CALCULATED CONVENTIONAL (y_2)
1,486	40	64.25	43.62
2,994	50	65.15	50.54
3,744	49	65.59	52.97
5,785	57	66.79	58.04
7,847	55	67.97	61.88
10,477	70	69.45	65.76
13,464	80	71.10	69.32
16,144	74	72.55	72.01
19,143	80	74.13	74.64
21,523	84	75.36	76.50
24,372	81	76.80	78.52
27,688	90	78.43	80.66
31,038	90	80.04	82.62
34,488	94	81.66	84.47
37,638	86	83.09	86.03
40,994	91	84.58	87.59
44,156	88	85.95	88.97
47,484	89	87.36	90.34
51,484	90	89.00	91.89
55,565	91	90.62	93.37
58,770	88	91.85	94.48
61,130	90	92.74	95.26
65,358	95	94.30	96.61
69,467	93	95.76	97.86
73,233	101	97.06	98.95
75,814	97	97.92	99.67
78,784	99	98.90	100.48
82,454	99	100.08	101.45
86,164	100	101.23	102.39
89,844	101	102.34	103.29
93,609	104	103.45	104.19
97,196	102	104.47	105.01
100,991	102	105.52	105.86
104,792	104	106.54	106.69

TABLE 7

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 7

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.298+.411\times10^{-5}x_i)}\right]$	0.848
Conventional	$y_2 = 1.031x_i^{.349}$	0.924

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
10,342	23	31.76	25.78
12,740	25	32.53	27.73
22,588	43	35.60	33.85
43,540	43	41.74	42.56
50,544	52	43.68	44.83
70,516	54	48.90	50.35
79,968	52	51.23	52.61
91,275	50	53.90	55.09
96,380	49	55.06	56.15
101,763	57	56.27	57.22

TABLE 8

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 8

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.209+.286\times 10^{-4}x_i)}\right]$	0.952
Conventional	$y_2 = .691x_i^{.442}$	0.984

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
2,481	19	26.88	21.97
9,395	42	41.80	39.58
14,311	56	50.76	47.67
20,834	59	60.85	56.28
32,369	66	74.67	68.39
44,103	78	84.75	78.41
62,343	98	95.02	91.37
79,043	106	100.71	101.48
96,923	104	104.43	111.05
111,783	104	106.36	118.28

TABLE 9

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 9

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.335+.177\times10^{-4}x_i)}\right]$	0.954
Conventional	$y_2 = 1.297x_i^{.379}$	0.985

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
5,199	33	38.21	33.00
9,415	39	43.39	41.33
13,387	47	47.92	47.23
15,878	52	50.61	50.38
18,548	55	53.35	53.44
22,439	58	57.13	57.43
28,948	67	62.90	63.25
35,066	63	67.75	68.02
39,421	70	70.89	71.11
42,267	75	72.82	73.01
47,444	83	76.08	76.28
51,212	79	78.27	78.52
55,049	83	80.36	80.70
58,771	81	82.26	82.72
62,651	77	84.10	84.75

TABLE 10

INDIVIDUAL OPERATOR DATA
OPERATOR NUMBER 10

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.316+.697\times10^{-5}x_i)}\right]$	0.959
Conventional	$y_2 = 3.004x_i^{.271}$	0.951

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
2,779	28	31.31	25.71
9,249	35	34.78	35.63
19,128	45	39.78	43.39
31,579	51	45.62	49.71
43,088	48	50.58	54.09
51,219	44	53.85	56.69
63,085	55	58.30	59.98
79,457	69	63.87	63.86
94,687	65	68.52	66.97
110,931	77	72.95	69.91
123,919	81	76.16	72.04
130,600	76	77.70	73.07
148,377	78	81.46	75.65

TABLE 11

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 11

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.593+.181\times10^{-4}x_i)}\right]$	0.658
Conventional	$y_2 = 4.527x_i^{.280}$	0.929

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
497	20	49.75	25.66
1,369	40	50.69	34.07
4,414	47	53.87	47.28
6,294	58	55.75	52.22
8,278	62	57.66	56.38
10,701	60	59.90	60.58
13,529	70	62.40	64.69
16,986	73	65.29	68.94
20,095	86	67.73	72.26
22,669	75	69.65	74.74
32,866	70	76.45	82.93
42,108	70	81.61	88.89

TABLE 12

INDIVIDUAL OPERATOR DATA
OPERATOR NUMBER 12

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.388+.281\times10^{-4}x_i)}\right]$	0.930
Conventional	$y_2 = 1.954x_i^{.362}$	0.981

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
433	16	36.24	17.56
1,973	34	39.37	30.40
3,929	41	43.14	39.02
5,665	44	46.32	44.54
6,925	41	48.54	47.90
8,466	49	51.14	51.52
10,986	63	55.17	56.61
13,470	66	58.86	60.95
16,878	72	63.53	66.14
21,237	75	68.89	71.87
25,197	79	73.21	76.46
34,115	79	81.37	85.33
46,868	85	89.99	95.72

TABLE 13

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 13

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.522+.237\times10^{-4}x_i)}\right]$	0.916
Conventional	$y_2 = 6.488x_i^{.244}$	0.993

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
96	20	44.86	19.65
1,819	39	47.46	40.25
3,995	49	50.60	48.75
6,483	52	54.00	54.86
7,851	57	55.78	57.48
10,121	60	58.62	61.15
13,188	63	62.22	65.23
15,862	70	65.15	68.23
19,457	77	68.80	71.71
21,892	82	71.11	73.80
25,183	77	74.03	76.37
40,881	82	85.19	85.94
57,736	90	93.35	93.48

TABLE 14

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 14

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.512+.536\times10^{-4}x_i)}\right]$	0.951
Conventional	$y_2 = 3.320x_i^{.330}$	0.992

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
48	11	44.23	11.90
1,682	43	49.75	38.50
4,563	50	58.37	53.52
7,969	68	66.99	64.33
10,409	74	72.26	70.26
13,644	81	78.27	76.82
17,833	87	84.65	83.92
21,577	98	89.26	89.36
26,354	99	93.95	95.46
30,946	99	97.45	100.66
35,776	100	100.31	105.59
40,397	100	102.44	109.91
48,067	103	104.99	116.40

TABLE 15

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 15

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.539+.190\times10^{-4}x_i)}\right]$	0.898
Conventional	$y_2 = 4.221x_i^{.278}$	0.961

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
2,324	35	48.58	36.25
9,738	54	56.65	53.97
15,804	71	62.45	61.75
21,855	64	67.61	67.57
30,063	67	73.73	73.83
38,038	89	78.83	78.81
41,331	72	80.71	80.65
46,168	87	83.28	83.17
56,067	92	87.86	87.79
65,696	96	91.56	91.74
71,821	91	93.58	94.04
80,781	91	96.15	97.16

TABLE 16

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 16

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110 \left[1 - e^{-(.220+.134\times10^{-4}x_i)}\right]$	0.987
Conventional	$y_2 = 1.733x_i^{.337}$	0.984

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
		CALCULATED	
	ACTUAL (y_i)	EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
2,318	25	24.43	23.42
13,712	45	36.57	42.61
27,350	49	48.85	53.76
34,873	51	54.72	58.34
46,325	64	62.60	64.19
60,617	66	70.87	70.27
75,977	70	78.16	75.83
94,757	87	85.25	81.68
108,917	93	89.54	85.60
126,317	95	93.80	89.98
137,009	96	95.96	92.47
155,129	98	98.99	96.42

TABLE 17

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 17

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.323+.190\times10^{-4}x_i)}\right]$	0.874
Conventional	$y_2 = 1.553x_i^{.365}$	0.951

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
	ACTUAL (y_i)	CALCULATED	
		EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
752	14	31.47	17.48
2,181	29	33.58	25.79
3,656	29	35.69	31.14
5,906	47	38.79	37.11
7,633	46	41.09	40.75
9,978	49	44.09	44.94
11,274	43	45.69	46.99
14,650	55	49.69	51.70
18,338	63	53.77	56.12
21,391	63	56.93	59.37
23,927	63	59.43	61.85
26,407	55	61.75	64.12
29,028	55	64.10	66.37
31,000	64	65.78	67.98
33,772	68	68.05	70.14

TABLE 18

INDIVIDUAL OPERATOR DATA

OPERATOR NUMBER 18

CURVES OF LEAST SQUARES FIT

TYPE OF LEARNING CURVE	EQUATION	COEFFICIENT OF CORRELATION (R)
Exponential	$y_1 = 110\left[1 - e^{-(.245+.693\times10^{-5}x_i)}\right]$	0.826
Conventional	$y_2 = 7.171x_i^{.183}$	0.546

CUMULATIVE PRODUCTION COUNT (x_i)	PER CENT EFFICIENCIES		
	ACTUAL (y_i)	CALCULATED	
		EXPONENTIAL (y_1)	CONVENTIONAL (y_2)
3,153	37	25.74	31.50
20,833	56	35.45	44.53
41,953	50	45.61	50.63
58,131	53	52.44	53.75
61,811	41	53.89	54.36
62,071	29	53.99	54.40
67,581	41	56.09	55.25
77,017	41	59.50	56.59
93,166	74	64.85	58.60
115,362	77	71.29	60.94
126,971	79	74.28	62.02
143,057	81	78.05	63.40
157,282	80	81.05	64.51

TABLE 19

SUMMARY OF COEFFICIENTS OF CORRELATION AND THEIR DIFFERENCES

1	2	3	4
	COEFFICIENTS OF CORRELATION		
OPERATOR	CONVENTIONAL	EXPONENTIAL	DIFFERENCES OF COEFFICIENTS (2 - 3)
1	.895	.782	+.113
2	.930	.819	+.111
3	.968	.847	+.121
4	.900	.647	+.253
5	.964	.921	+.043
6	.969	.948	+.021
7	.924	.848	+.076
8	.984	.952	+.032
9	.985	.954	+.031
10	.951	.959	-.008
11	.929	.658	+.271
12	.981	.930	+.051
13	.993	.916	+.077
14	.992	.951	+.041
15	.961	.898	+.063
16	.984	.987	-.003
17	.951	.874	+.077
18	.546	.826	-.280

TABLE 20

OPERATION ASSIGNMENTS

OPERATORS NUMBER 1 THROUGH 6

OPERATION	A	B	C	D	E
OPERATOR					
1				X	
2	X				
3			X		
4			X		
5	X				
6	X				
Cycle Time (BH/C*)	1.0589	0.7052	0.4863	0.3809	0.7152
Manual Effort (%)	100	100	50	75	75

*BH/C is Base Hours Per 100 Units

TABLE 21

OPERATIONS ASSIGNMENTS

OPERATORS NUMBER 7 THROUGH 18

OPRN	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
OPR																			
7			X			X		X	X										
8						X	X	X	X						X		X		
9			X						X										X
10			X				X	X	X				X			X		X	
11			X				X	X	X				X					X	
12			X																
13			X																
14			X				X	X											
15				X	X		X	X	X	X								X	
16							X	X											
17			X			X	X	X											
18	X	X				X	X	X	X		X	X		X				X	
BH/C	.034	.001	.821	.345	.241	.248	.055	.194	.316	.033	.069	.118	.073	.065	.048	.126	.165	.280	.363
%	0	100	100	100	100	100	100	100	100	100	0	50	100	80	100	100	100	100	100

NOTE: Operators number 7 and 8 were assigned to the first work shift; all others were assigned to the third work shift.

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